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A numerical study of boundary-layer heat and mass transfer in a forced flow of humid air with surface steam condensation

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Abstract

Data obtained by numerical solution of boundary-layer flow, energy and diffusion equations for laminar and turbulent flows of humid air are described. The effect of vapor concentration and vapor–gas mixture temperature on the intensity of and similarity between the heat- and mass-transfer processes in the presence of surface vapor condensation is considered. It is shown that the Reynolds analogy is valid in the range of mass vapor concentration in the flow core $C_{10} < 0.2$. In the region of higher vapor concentrations (condensation of a vapor with incondensable species), the analogy between the transfer phenomena is violated and the assumption that $Le = 1$ can no longer be used.
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1. Introduction

Ducted humid-air flows are often used in various technical applications. First of all, such applications include humidity control systems, air cleaning and conditioning systems, and also apparatus for drying materials in humid gases. If the temperature of the streamlined surface is lower than the saturation point, then the steam contained in the air undergoes condensation. In this case, the total wall heat flux includes the component due to convection, spent on cooling the vapor–gas mixture in the boundary layer, and the phase-transition heat, spent on vapor condensation.

In humid air, the mass concentrations of water vapor under moderate temperatures ($T_0 < 60$ °C) are rather low ($C_{10} < 0.1$). However, because the phase-transition heat is high, the above heat-flux components are comparable with each other and, depending on particular conditions, regimes with bright manifestation of convective heat transfer or with prevailing contribution due to the phase-transition heat can be obtained.

In humid air, surface condensation is determined both by the boundary-layer resistance to steam diffusion and by the thermal resistance of the boundary layer. That is why the heat- and mass-transfer processes are interrelated ones, presenting a complex phenomenon, which, to be adequately understood, necessitates joint solution of flow, energy and diffusion equations.

A key point in the problem of interest is whether it is possible or not to use the analogy between the heat- and mass-transfer phenomena under such conditions. Among many works experimentally treating the problem of heat and mass transfer in humid air, there is no unambiguous opinion concerning this matter. For instance, the data of [1–3] show the heat- and mass-transfer processes to be similar with the criterial relations for them being identical to the regularities observed for flows without phase transitions. On the other hand, the data of [4,5] show the analogy to be violated, the heat-transfer rate being well in excess of that in the case of a flow over a "dry wall".

The experimental data of [6] for the flow with steam condensation from humid air under natural-convection conditions on a vertical surface are also indicative of Reynolds-analogy violation. In contrast, the recently reported numerical study [7] of heat- and mass-transfer processes on a vertical plate has proved the similarity between the Sherwood number and the Nusselt number calculated from the convective component of the total wall heat flux.

The extensive experimental study [8] of convective heat transfer from humid air deserves special mention.

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Nomenclature

$b_{1d} = (C_{1w} - C_{10})/(1 - C_{1w})$ diffusion surface permeability parameter due to vapor condensation
C_1 mass concentration of vapor
Cp_1; Cp_2; $\overline{C}p$ specific heat capacities of water vapor, air, and gas mixture, J/(kg K)
$C_f/2$ friction
d moisture content (kg/kg)
$D_{1,2}$ vapor diffusivity (m^2/s)
H total enthalpy, $(\int Cp_i\,\mathrm{d}T + H_{oi})C_i$ (J/kg)
k turbulent kinetic energy (m^2/s^2)
Ku Kutateladze number, $r/(H_w - H_0)$
Le Lewis number, $\overline{C}p\rho D_{12}/\lambda$
n power exponent in the heat and mass transfer and friction laws
Pr Prandtl number, ν/a
r phase-transition heat (J/kg)
q_λ, q_{dif}, q_Σ, q_{H}, q_j, q_{w} wall heat flux components defined by formulas (3,4,5) and (11), (W/m^2)
Re_x Reynolds number calculated from the current coordinate, $u_0 \cdot x/\nu$
Re_{T} turbulent Reynolds number, $\rho k^2/\mu\tilde{\varepsilon}$
Sc Schmidt number, ν/D
St_d, St_λ, St_{w}, St_Σ, St_{H} Stanton numbers defined by formulas (6)–(10)
T temperature (K)
U, V longitudinal and transverse velocities (m/s)
x, y longitudinal and transverse coordinates

Greek symbols
ε dissipation rate of turbulent kinetic energy (m^2/s^3)
λ thermal conductivity of the vapor–gas mixture (W/(mk))
μ dynamic viscosity (Ns/m^2)
ν kinematic viscosity (m^2/s)
ρ density of the vapor–gas mixture (kg/m^3)
τ shear stress (N/m^2)

Subscripts
1 vapor
2 air
i gas-mixture component
j heat flux spent on the phase transition
O flow core
W wall parameters and wall heat flux
d, d_{if} diffusion-related quantity
λ convective heat flux
Σ total heat flux (in formulas (3) and (4))
τ friction rate

The authors of [8] found that the similarity between the heat- and mass-transfer processes is observed only in a very narrow range of the vapor-concentration difference between the flow core and the wet surface, whereas in the prevailing region of temperatures and vapor concentrations the analogy does not hold.

The analogy between the heat- and mass-transfer processes, if valid, substantially facilitates the solution of the problem under study. Such an analysis, based on solving integral energy and diffusion relations, was performed in [9–11]. The similarity between the heat- and mass-transfer processes, laid to the basis of these studies, is supported by good agreement between the calculation results and the experimental data of [12] for pressurized air flow.

The boundary-layer heat and mass transfer results in a non-uniform distribution of temperatures and concentrations of components across the boundary layer, which in turn causes variation of Prandtl, Schmidt and Lewis numbers. Apparently, these effects are hard to be allowed for in the framework of the integral approach; for this reason, it becomes necessary to solve the full system of differential boundary-layer energy and diffusion equations. The present work is aimed at a numerical study of boundary-layer transfer processes with surface steam condensation from humid air and determination of the domain where the triple Reynolds analogy for these conditions holds.

For the laminar flow regime, the similarity or dissimilarity between the transfer processes can be established theoretically since the formulation of the problem of interest and its solution do not require invoking any additional hypotheses or simplifying assumptions.

In the present paper, the turbulent regime of the humid-air flow was also addressed. To model the turbulence, the k–ε model of turbulence was used. The solution of this problem is of special significance for practical applications, and, to verify it, we compared the calculation results with available experimental data.

With increasing temperature of the steam–air mixture, the steam concentration under saturation also increases, while the air concentration decreases. In this case, the problem about heat and mass transfer in humid air reduces to the practically important problem about vapor condensation in the presence of an incondensable impurity. This problem, taken alone, is also of substantial interest; it was addressed in many experimental and theoretical studies and therefore is not discussed in detail in the present work.

2. Problem statement: governing equations and numerical algorithm

The schematic diagram of the problem is shown in Fig. 1; the same figure indicates the main adopted notations. On the whole, they are analogous to those used in [13]. We consider the flow of a binary steam–air mixture over an infinite flat surface. The flow temperature is T_0 and the mass concentration of steam in the flow is C_{10}. The moisture content d_0 and the relative humidity φ_0 in the flow core are constant along the duct length.

The steam–air mixture can be either in the saturated ($\varphi_0 = 100\%$) or in the overheated ($\varphi_0 < 100\%$) state. As a limiting test case, heat transfer and friction in a dry-air flow were calculated ($\varphi_0 \to 0$).

Since the present work was aimed at studying the heat- and mass-transfer processes in the boundary-layer flow of the vapor–gas mixture, to simplify the problem, we ignore the effect due to the condensate film formed on the duct wall, assuming its thermal resistance to be small. This assumption is well justified by the fact that, in a number of experimental heat- and mass-transfer studies of humid-air flows (see, for instance, [8]), measurements of the film-surface temperature were performed, which makes it possible to consider the transfer processes in the vapor–gas boundary-layer flow individually, irrespective of the condensate-film flow. Then, the surface temperature assumes the value equal to the saturation temperature $T_w = T^*$ at the given vapor concentration C_{1w} at the duct wall.

The continuity, momentum, vapor-diffusion, and energy equations are

$$
\begin{aligned}
&\frac{\partial(\rho U)}{\partial x}+\frac{\partial(\rho V)}{\partial y}=0,\\
&\rho U\frac{\partial U}{\partial x}+\rho V\frac{\partial U}{\partial y}=\frac{\partial}{\partial y}\left[(\mu+\mu_T)\frac{\partial U}{\partial y}\right],\\
&\rho U\frac{\partial C_1}{\partial x}+\rho V\frac{\partial C_1}{\partial y}=\frac{\partial}{\partial y}\left[\left(\frac{\mu}{Sc}+\frac{\mu_T}{Sc_T}\right)\frac{\partial C_1}{\partial y}\right],\\
&\rho\overline{C}p\left(U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left[\overline{C}p\left(\frac{\mu}{Pr}+\frac{\mu_T}{Pr_T}\right)\cdot\frac{\partial T}{\partial y}\right]\\
&\qquad+\rho D_{12}(Cp_1-Cp_2)\frac{\partial C_1}{\partial y}\cdot\frac{\partial T}{\partial y},
\end{aligned}
\tag{1}
$$

where ρ and $\overline{C}_p$ are the density and heat capacity of the binary gas mixture, U and V are the x- and y-velocities, μ, and μ_T are the molecular and turbulent viscosities, and Pr, Sc, and Le are the Prandtl, Schmidt, and Lewis numbers; the subscripts 1 and 2 refer to steam and air, respectively.

To model the turbulence, the k–ε model of turbulence (LRN-modification proposed by Chien [14]) was used.

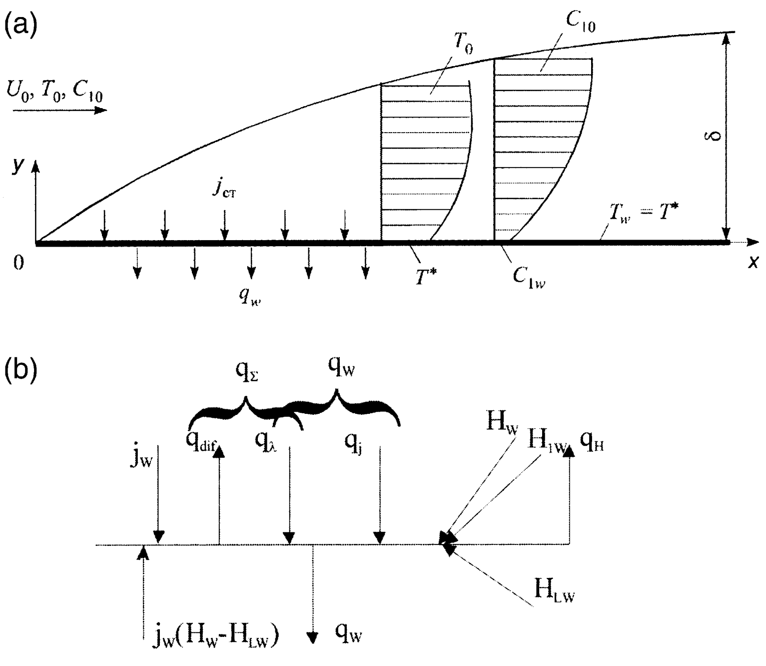

Fig. 1. (a) Diagram of the boundary-layer flow with steam condensation from humid air. (b) Ratio of the wall heat fluxes under humid-air condensation conditions.

$$\rho U\frac{\partial k}{\partial x}+\rho V\frac{\partial k}{\partial y}=\frac{\partial}{\partial y}\left[(\mu+\mu_{\mathrm{T}})\frac{\partial k}{\partial y}\right]+\mu_{\mathrm{T}}\left(\frac{\partial U^2}{\partial y}\right)-\rho\varepsilon,$$

$$\rho U\frac{\partial\tilde{\varepsilon}}{\partial x}+\rho V\frac{\partial\tilde{\varepsilon}}{\partial y}=\frac{\partial}{\partial y}\left[\left(\mu+\frac{\mu_{\mathrm{T}}}{1.3}\right)\frac{\partial\tilde{\varepsilon}}{\partial y}\right]+\left[c_1 f_1\mu_{\mathrm{T}}\left(\frac{\partial U}{\partial y}\right)^2-\rho c_2 f_2\tilde{\varepsilon}\right]\frac{\tilde{\varepsilon}}{k}-2\mu\frac{\tilde{\varepsilon}}{y^2}\mathrm{e}^{-0.5y^+}, \tag{2}$$

where $f_1 = 1$, $f_2 = 1 - 0.22\mathrm{e}^{-(\frac{Re_{\mathrm{T}}}{6})^2}$, $\varepsilon = \tilde{\varepsilon} + 2v\frac{k}{y^2}$, $c_1 = 1.35$, $c_2 = 1.8$, $\mu_{\mathrm{T}} = c_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}$, $y^+ = \frac{yu_\tau\rho}{\mu}$, $Re_{\mathrm{T}} = \frac{\rho k^2}{\mu\tilde{\varepsilon}}$, and $f_\mu = 1 - \mathrm{e}^{-0.0115y^+}$. Here k is the kinetic turbulence energy and ε is the dissipation rate of this energy.

The turbulent Prandtl and Schmidt numbers were assumed constant across the boundary layer and equal to 0.9.

The boundary conditions were as follows:

(1) at the inlet cross-section of the duct ($x = 0$): $k = k_0(y)$, $C_1 = C_{10}(y)$, $U = U_0(y)$, $V = V_0(y)$, $\varepsilon = \varepsilon_0(y)$, and $T = T_0(y)$.
(2) at the outer border of the boundary layer ($y = \delta$): $U = U_0$, $T = T_0$, $C_1 = C_{10}$, and $\partial\phi/\partial y = 0$ for all other variables.
(3) at the wall ($y = 0$): $U = 0$, $k = 0$, $\varepsilon = 0$, $C_1 = C_{1\mathrm{w}}$, $T = T_{\mathrm{w}}$, and $\rho V = \frac{\rho D}{1 - C_{1\mathrm{w}}}\left(\frac{\partial C_1}{\partial y}\right)_{\mathrm{w}}$.

The thermal and physical properties of air and water vapor were calculated by polynomials taken from [15]. To solve the resultant system of equations, the finite-difference scheme described in detail in [16] was used.

In solving the system of (1) and (2), as input data, the outside-flow parameters (temperature, air humidity, and velocity), and also the wall temperature $T_{\mathrm{w}} = \mathrm{const}$ and the steam concentration $C_{1\mathrm{w}} = \mathrm{const}$ corresponding to the saturation curve were set.

3. Heat balance under surface steam condensation conditions

For a gas mixture whose one component (vapor) undergoes surface condensation, which is equivalent to a boundary layer with porous suction, the heat flux is carried over via thermal conduction $q_\lambda = -\lambda\frac{\partial T}{\partial y}$ and diffusion $q_{\mathrm{dif}} = \sum j_i H_i$ (Fig. 1b). Here j_i, is the flux of the ith substance and $H_i = (\int Cp_i\,\mathrm{d}T + H_{oi})C_i$ is its total enthalpy (H_{oi} is the formation enthalpy of the ith substance).

For a binary gas mixture, the following expression for the total heat flux due to thermal conduction and diffusion can readily be obtained [17]:

$$q_\Sigma = -\lambda\left(\frac{\partial T}{\partial y}\right)_{\mathrm{w}} + j_{\mathrm{w}}(H_{1\mathrm{w}} - H_{\mathrm{w}}), \tag{3}$$

where H_{w} and $H_{1\mathrm{w}}$ are the total enthalpy of the gas mixture and that of the steam at the wall.

The thermal balance on the wall is given by the following equation of heat conservation on permeable surface:

$$q_{\mathrm{w}} = q_\Sigma + j_{\mathrm{w}}(H_{\mathrm{w}} - H_{\mathrm{LW}}). \tag{4}$$

Here q_{w} is the total heat flux withdrawn from the wall and H_{Lw} is the total enthalpy of the condensed liquid. Combining (3) and (4), we obtain the following relation for the wall heat balance:

$$q_{\mathrm{w}} = -\left(\lambda\frac{\partial T}{\partial y}\right)_{\mathrm{w}} + j_{\mathrm{w}}\cdot r, \tag{5}$$

where r is the phase-transition heat. Thus, the total heat-flux density withdrawn from the wall with steam condensation can be obtained as the sum of the convective component and the thermal energy spent on the phase transition.

The main series of the present heat- and mass-transfer calculations was performed with variation of vapor–gas mixture parameters in the flow core (T_0 = 40–60 °C, C_{10} = 0–0.2) and wall temperature (T_{w} = 10–40 °C) under atmospheric pressure. These conditions comply with those in the majority of experimental studies. In addition, to find the boundaries of the region in which the similarity between heat- and mass-transfer processes is valid, calculations for higher temperatures ($T_0 \leqslant 100$ °C) and steam concentrations ($C_{10} \leqslant 1$) in the flow core were performed; under these conditions, the steam–air mixture approaches the limiting state of pure saturated vapor flow. The temperature and moisture content in the flow and on the wall remained unchanged over the duct length. The Reynolds number for the laminar and turbulent flow regimes was varied respectively in the ranges $Re_x = 10^3$–5×10^4 and 5×10^4–10^6.

The dependence of wall heat-flux components at a fixed steam concentration in the core of the humid-air flow ($C_{10} = 0.04$) for various temperature differences between the flow core and the wall is shown in Fig. 2. The data for the laminar ($Re_x = 10^4$) and turbulent ($Re_x = 10^6$) flow regime are shown in Fig. 2a and b, respectively. Positive and negative values of q_i correspond to the heat withdrawal from the wall and to the heat supply to the wall, respectively.

As it is evident from Fig. 2, the data for the laminar and turbulent flow regimes exhibit a similar behavior. With increasing temperature difference $\Delta T = T_0 - T_{\mathrm{w}}$, the convective heat-flux component q_λ increases; as a result, the heat flux q_{w} withdrawn from the wall also increases. The diffusion component q_{dif} decreases with increasing flow-core temperature, together with the thermal energy $q_j = j_{\mathrm{w}}\cdot r$ spent on the phase transition during steam condensation. The heat transfer due to diffusion attains its highest value, which can be well in

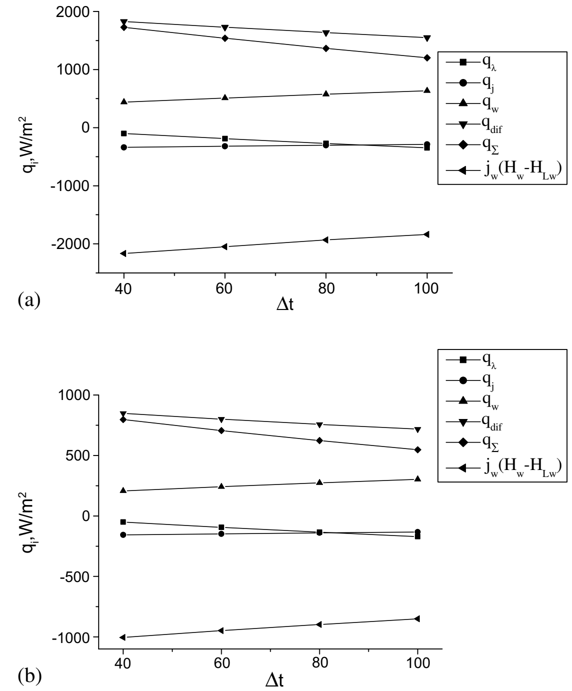

Fig. 2. Heat fluxes on the surface with humid-air condensation. $T_w = 20$ °C, $C_{10} = 0.04$; (a) $Re_x = 10^4$; (b) 10^6.

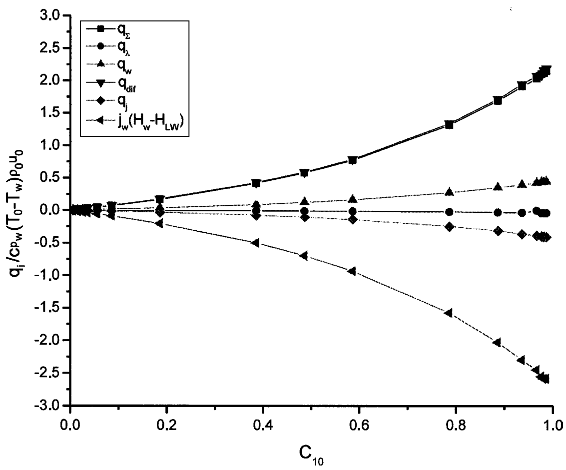

Fig. 3. Effect of concentration difference on the heat-flux components. $T_0 = 100$ °C, $T_w = 20$ °C, $Re_x = 10^4$.

excess of both the convective component and the energy spent on the phase transition. The diffusion (q_{dif}) and convective (q_λ) components are oppositely directed, so that the total convective heat flux q_Σ is directed from the surface and, together with the heat q_w withdrawn from the wall, is compensated by the flux of thermal energy $j_w(H_w - H_{LW})$ at the interface between the phases (see formula (4)).

The effect of steam concentration on the proportion between the heat-flux components for the fixed temperature difference $\Delta T = 80°$ ($T_0 = 100$ °C, $T_w = 20$ °C) is illustrated by Fig. 3. As the steam concentration increases, all the wall heat-flux components increase without exception. Like in Fig. 2, the diffusion heat flux is a prevailing one and, as the vapor–air mixture approaches the state of pure saturated vapor, the diffusion component may be one order of magnitude greater than the heat transfer due to molecular or turbulent thermal conduction.

The central point in presentation of calculation results for the combined heat- and mass-transfer problem under consideration in the criterial form is the choice of governing criteria for friction and heat and mass transfer. For friction and mass transfer, these criteria are unambiguous:

$$\frac{Cf}{2} = \frac{\tau_w}{\rho_0 u_0^2} : St_d = -\left(\rho D_{12}\frac{\partial C_1}{\partial y}\right) \Big/ \rho_0 u_0 \Delta C_1. \quad (6)$$

For heat transfer, under conditions in which several heat-flux components act upon the wall, different forms of the heat-transfer coefficient can be adopted, the main ones being the following:

The Stanton number calculated from the convective heat-flux component:

$$St_\lambda = \left(-\lambda\frac{\partial T}{\partial y}\right)_w \Big/ \rho_0 u_0 \overline{C}p_0 (T_w - T_0), \quad (7)$$

the Stanton number calculated from the heat flux withdrawn from the wall:

$$St_w = \left[\left(-\lambda\frac{\partial T}{\partial y}\right)_w + j_w \cdot r\right] \Big/ \rho_0 u_0 \overline{C}p_0 (T_w - T_0), \quad (8)$$

the Stanton number calculated from the heat flux due to convection and diffusion:

$$St_\Sigma = (q_\Sigma)_w / \rho_0 u_0 (H_w - H_0) \quad (9)$$

and the Stanton number calculated from the total-enthalpy gradient at the wall:

$$St_H = q_H / \rho_0 u_0 (H_w - H_0), \quad (10)$$

where

$$q_H = \left(-\frac{\lambda}{\overline{C}p}\cdot\frac{dH}{\partial y}\right)_w. \quad (11)$$

Particular forms (8)–(10) of Stanton number depend both on the choice of transfer parameters (temperature and total enthalpy) and on the heat-flux component defining, the convective heat flux, the total heat flux including that due to diffusion, or the total heat flux withdrawn from the wall. A detailed analysis of the relation between the different forms of thermal Stanton number is given in [17] for the case of porous blowing of foreign gases into boundary layer. For suction or condensation of vapor–gas mixtures, no similar analysis has been so far reported.

Since, in the majority of humid-air condensation experiments, measured parameters were the total heat flux withdrawn from the wall (q_w) and the energy spent on the phase transition ($q_j = j_w \cdot r$), from which, using the wall heat balance relation (5), the convective component $q_\lambda = (-\lambda \frac{\partial T}{\partial y})_w$ can be calculated, it is these heat-flux components that will be given below most attention.

4. Calculation results: comparison with available experimental data

The predicted friction and rates of heat and mass transfer in the boundary layer for the laminar and turbulent regimes of humid-air flows with various moisture contents in the flow core are shown in Fig. 4. Solid lines 1 and 2 bound the convective heat-transfer region St_λ (7), the mass-transfer region, and the wall-friction region (formulas (6)). The dashed curves are the predicted total Stanton numbers St_w calculated from the heat withdrawn from the wall (by relation (8)). Curves 1 in this graph show the calculation data for dry air; they comply with the known regularities for laminar and turbulent flows [18]. The calculation results for friction and convective heat and mass transfer for various moisture contents in the flow fall into the shaded regions; curves 2 bounding these regions from above are obtained for the saturated state of the vapor–gas mixture.

The following specific features of the calculation data in Fig. 4 are worth noting. The steam concentration is seen to have only insignificant effect on the friction and heat- and mass-transfer coefficients, no greater than 10% for the conditions under consideration. The increase in the transfer coefficients is due to the boundary-layer suction through condensation; however, because the transverse flux of matter is small, it can be ignored for humid-air flows with moderate parameters.

The calculations showed that the friction coefficient and the convective heat and mass-transfer coefficients with allowance for the Prandtl and Schmidt number agree fairly well with each other both for the laminar and turbulent flow regimes. This means that, for the conditions under consideration, the triple Reynolds analogy holds:

$$c_f/2 = St_\lambda Pr^n = St_d Sc^n,$$

where the power exponent is $n = 0.66$ and $n = 0.6$ for the laminar and turbulent flow regimes.

Curves 3 in Fig. 4 shows the calculation data for the total heat transfer obtained by formula (8) for various steam concentrations with allowance for the phase-transition heat. The surface steam condensation is seen to lead to substantial heat-transfer intensification. For instance, for the steam concentration in the flow core $C_{10} \approx 5\%$, the heat transfer on the surface is increased by more than fivefold compared to the case of dry-air flow. Here, the computed curves for various steam concentrations are equidistant, being indicative of insignificant influence of flow Reynolds number on the degree of heat-transfer intensification due to the phase transition both in the laminar and turbulent boundary layers.

The proportion between the convective heat flux and the heat flux withdrawn from the wall is determined by the concentration difference ΔC_1, and the temperature difference between the flow core and the wall. This follows from Fig. 5 that shows the calculated convective heat-flux component q_λ/q_w versus the steam concentration difference $\Delta C_1 = C_{10} - C_{1w}$ for various temperatures of the steam–air mixture. For the sake of convenience, the calculated values are shown as open

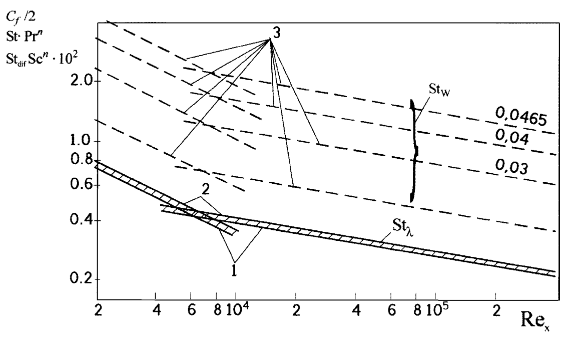

Fig. 4. Heat and mass transfer, and friction of laminar and turbulent humid-air flows. $T_0 = 40$ °C, $T_w = 20$ °C. Curves 1 and 2—friction and heat and mass transfer in dry and humid ($C_{10} = 0.0465$) air, respectively; the shaded area is the region of moderate moisture contents. Curve 3—total wall heat flux for various steam concentrations.

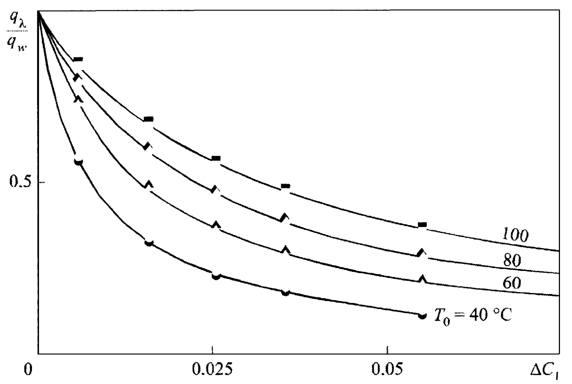

Fig. 5. Effect of concentration difference on the convective heat-flux component. $T_w = 20$ °C. The open and full circles show the data for the laminar and turbulent flow, respectively. Curves—calculation by formula (12).

and full circles respectively for the laminar and turbulent flow regimes.

The most important conclusion from Fig. 5 is that the predicted q_λ/q_w ratios for the laminar and turbulent flow regimes are almost identical, showing these heat-flux components to be independent of the flow regime.

For comparison, the same figure shows the data calculated by the following formula [10]:

$$(q_\lambda/q_w)_w = 1 + Le^n b_{1d} Ku, \tag{12}$$

this formula was derived based on the balance relations for the wall heat and mass fluxes, and also on the similarity between the concentration and enthalpy fields of the vapor–gas mixture. Here

$$b_{1d} = j_w/\rho_0 U_0 St_d = (C_{10} - C_{1w})/(1 - C_{1w}) \tag{13}$$

is the diffusion permeability parameter and

$$Ku = r/(H_0 - H_w) \tag{14}$$

is the Kutateladze number.

Fig. 5 shows that the values calculated by formula (12) agree well with the numerical solution of full equations for laminar and turbulent boundary-layer flows in the whole range of vapor concentrations and temperatures. The numerical data obtained for other Reynolds numbers and various wall temperatures exhibit a similar behavior.

The universal character of relation (12), and also the weak dependence of convective heat-transfer coefficients on the humid-air parameters, evident from Fig. 4, gives ground for the following conclusion important from the practical point of view. The relative values of the wall heat-flux components in humid-air flows with condensation can be calculated by formula (12) irrespective of the flow regime. The values of convective heat- and mass-transfer coefficients can be found by solving the integral boundary-layer relations; as a first approximation, the dry-wall regularities can be used. This approach allows substantial simplification of the numerical procedure compared to the solution of the system of differential boundary-layer flow equations.

Strictly speaking, the above conclusions are valid for the steam–air system. Apparently, for other vapor–gas system, an additional analysis is necessary. The same can be stated concerning the range of applicability of Reynolds analogy to steam–gas mixtures with higher steam concentrations ($C_{10} \to 1$), i.e., for steam with admixed incondensable gas species.

Fig. 6 shows variation of the coefficients of Reynolds analogy between the heat transfer, the mass transfer, and the friction with allowance for the standard corrections applied for Prandtl and Lewis numbers. The calculations were performed for the fixed wall temperature $T_w = 20$ °C and various temperatures and concentrations of steam in the undisturbed flow up to the limiting value of $C_{10} \to 1$ corresponding to pure water vapor at the temperature $T_0 = 100$ °C. Thus, in the present study we analyzed the widest possible range of steam concentrations in humid air to reveal the boundaries of the region in which the Reynolds analogy holds.

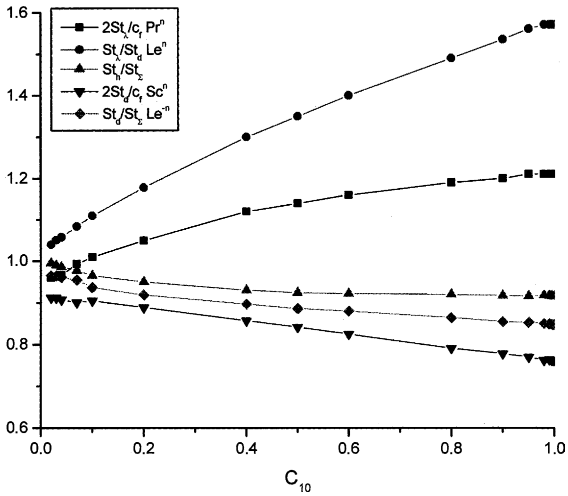

Fig. 6. Reynolds-analogy coefficients on the surface with steam condensation. $T_0 = 100$ °C, $T_w = 20$ °C, $Re_x = 10^4$.

The numerical analysis was performed both for the laminar and turbulent flow regimes. The calculated values turned out to be almost identical provided that, in the calculations, the Prandtl and Lewis numbers are used with their respective power exponents ($n = 0.66$ and $n = 0.6$ for the laminar and turbulent flow regimes, respectively).

As it is seen from Fig. 6, the factor of dissimilarity increases with increasing steam concentration. This can be explained by the fact that, as the steam concentration in the mixture increases, the Prandtl and Lewis numbers also increase. Here, the degree of dissimilarity between the heat- and mass-transfer processes is much greater than between the mass transfer and the friction. This behavior of the relative transfer coefficients is determined, first of all, by the influence due to the diffusion heat transfer component, whose value, as it follows from energy equation (1), increases with increasing Le number. However, as the calculations show, the data in Fig. 6 in the region of high steam contents ($C_{10} \to 1$) are not universal ones, depending on the temperature difference ΔT between the flow core and the wall. That is why the solution of the problem about the analogy between the heat- and mass-transfer processes under such conditions requires special consideration.

The following important feature of Fig. 6, directly related to the humid-air flow, deserves special mention. For steam concentrations in the air flow $C_{10} < 0.2$, where the dissimilarity factor differs from unity insignificantly (10–15%), in practical calculations one can use the Reynolds analogy. To these concentrations, the saturation temperatures $T_0 \approx 70$–80 °C correspond, which constitute the upper boundary of the Reynolds analogy for humid air. For the temperature region $T_0 < 60$ °C, most frequently dealt with in technical applications, the triple-analogy coefficients are very close to unity.

Thus, to calculate friction and heat and mass transfer in the above temperature region, one can use the integral method, using only one transfer equation, as it was proposed in [10]. Doing this, one must take into account the effect of steam suction on the rates of heat and mass transfer in the laminar [18] and turbulent regime [19] of the vapor–gas flow.

The similarity between the heat- and mass-transfer processes established in the experimental studies [1–3,12] lends support to the results of the present numerical analysis. As it was shown in [10], the coincidence between the experimental data on the average heat and mass transfer with results yielded by the integral analogy and, hence, by the virtue of this analogy, also for the solution of the system of differential equation (1) in the full statement, can be considered as rather satisfactory (within 20%). The reason for such substantial deviation between the data can be inadequacy of the boundary conditions adopted in the calculations to those in the experiments, owing to variation of wall temperature and heat flux over the duct length in experimental studies [12].

The present numerical results were also compared with the experimental data of [8].

Although the law of mass transfer turned out to be conservative with respect to thermal and concentration boundary conditions and compliant with the classical regularities for turbulent flows, the convective heat transfer in the present study proved to be strongly dependent on the moisture content of the flow. The degree of dissimilarity between heat- and mass-transfer processes is substantial even for low steam concentrations in the air. Fig. 7 compares the experimental data of [8] with the present numerical results, showing the wall heat-flux components q_λ/q_w and q_j/q_w as functions of the difference between the steam concentrations in the flow core and at the wall. Although there is qualitative similarity between the data, the quantitative difference is rather substantial. For instance, the difference between the theoretical predictions and the experimental data judged by the convective component q_K/q_w amounts to more than threefold for the concentration differences $\Delta C = 0.04$–0.05. A very pronounced disagreement is also observed in the region of low moisture contents, and the coincidence between the theory and the experiment is close only in a narrow region of steam concentration around $\Delta C_1 \approx 0.023$.

The reason for the lack of the analogy between the heat- and mass-transfer processes in the experimental data of [8] still remains unclear. With it, also obscure is the physical mechanism resulting in the experimentally observed violation of the analogy at low steam concentrations in the boundary layer. One of possible reasons for this violation can be the presence of the liquid film on the wall and/or generation of waves in it differently affecting the heat and mass transfer with the vapor–gas mixture. In the present study, these effects were neglected. Apparently, to clarify this point, further experimental studies with various thermal and concentration boundary conditions for different flow regimes and geometries (developing boundary layer, stabilized ducted flow) are necessary.

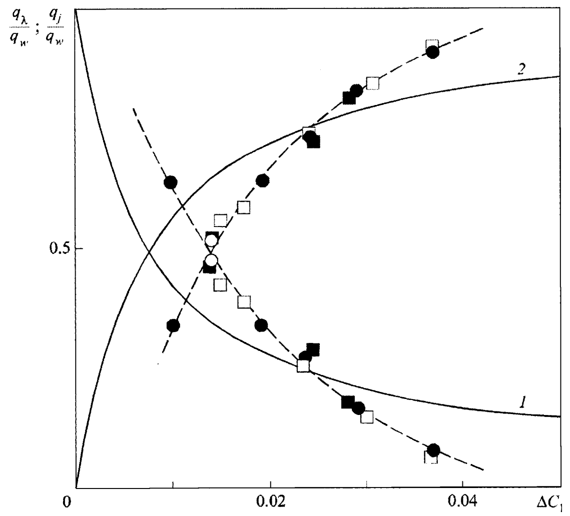

Fig. 7. Comparison of predicted and measured heat fluxes for humid air. $T_0 = 40$ °C, $T_w = 21$ °C. Circles—experimental data of [8]. Curves 1 and 2—predicted and ratios. q_λ/q_w and q_j/q_w ratios.

5. Conclusions

The numerical solution of the system of differential energy, diffusion, and boundary-layer flow equations for the laminar and turbulent flows of humid air with surface steam condensation shows that the momentum- and matter-transfer processes and the convective heat transfer are similar with allowance for the standard corrections for the Prandtl and Lewis numbers in the range of steam concentrations in the flow core $0 < C_{10} \leqslant 0.2$. Heat and mass transfer in humid air with such steam concentrations obeys the "dry-wall" regularities. The total heat transfer on the wall can be substantially increases due to the phase-transition heat; with increasing moisture content, the heat-transfer intensification ratio also increases.

The calculations showed that the relative convective heatflux component g_λ/q_w and the heat q_j/q_w released due to steam condensation are practically independent of the particular flow regime and coincide with the relations obtained from the balance equations of energy and mass conservation at the wall with invoking the condition of similarity between the heat- and mass-transfer processes. This substantiates the possibility of using the integral methods for calculating heat- and mass-transfer processes in laminar and turbulent ducted humid-air flows.

On the whole, the above conclusions are supported by the experimental data of [1–3,9] and disagree with those of [8].

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